№1.

$$(y')^2 = y''(y+5)$$

$y' = p(y)$

$y'' = p'(y) \cdot y' = p'(y) p$

1. $p^2 = p' p (y+5) \quad | : p (\neq 0)$

$p = p'(y+5)$

$$\frac{1}{y+5} = \frac{dp}{dy} \cdot \frac{1}{p}$$

$$\frac{dy}{y+5} = \frac{dp}{p}$$

$\ln|y+5| = \ln p + \ln c_1$

$y+5 = C_1 p$

$y+5 = \frac{dy}{dx} C_1$

$$\frac{dx}{C_1} = \frac{dy}{y+5}$$

$C_1^{-1} x = \ln|y+5| + \ln|C_2| = \ln|C_2(y+5)|$

$$\therefore \quad y = \frac{e^{\frac{x}{C_1}}}{C_2} - 5$$

2. $p = 0$

$y = C_3$

Проверка: $(C_3')^2 = 0$

$C_3''(C_3+5) = 0$

$y=C_3$ - решение

$y_1 = \frac{e^{\frac{x}{C_1}}}{C_2} + 5$

$y_2 = C_3$

52

$(y''-2x)\,x\cdot\sin x + (y'-x^2)(x\cos x - \sin x) = 0$

$p = y' - x^2$

$p' = y'' - 2x$

$p'\,x\cdot\sin x = p(\sin x - x\cos x)$

Делить как линейное!

$\frac{dp}{dx}\,\frac{1}{p} = \frac{\sin x - x\cos x}{x\cdot\sin x}$

$\frac{dp}{p} = \left(\frac{1}{x} - ctg x\right)dx$

$\ln|p| = \ln|x| - \ln|\sin x| + \ln|C_1|$

$p = \frac{xC_1}{\sin x}$

$y' = \frac{xC_1}{\sin x} + x^2$

$dy = \left(\frac{xC_1}{\sin x} + x^2\right)dx$

$y = \frac{x^3}{3} + C_1\int\frac{x}{\sin x}dx + C_2$

§3

$$6y'' - y' - y = \cos\frac{x}{2} + e^{\frac{x}{2}}$$

1) $6y'' - y' - y = 0$

$6k^2 - k - 1 = 0$

$D = 1 + 24 = 25$

$k_{1,2} = \frac{1 \pm 5}{12}$ $\quad k_1 = -\frac{1}{3}$ $\quad k_2 = \frac{1}{2}$

$y_{o.o} = C_1 e^{-\frac{1}{3}x} + C_2 e^{\frac{1}{2}x}$

2) $f_1 = \cos\frac{x}{2} = e^0\left(1\cdot\cos\frac{x}{2} + 0\cdot\sin\frac{x}{2}\right)$

$\alpha = 0, \ \beta = \frac{1}{2}$

$m = 0 \quad n = 0$

$y_{чн_1} = A_1\cos\frac{x}{2} + B_1\sin\frac{x}{2}$

$y'_{чн_1} = -\frac{A_1}{2}\sin\frac{x}{2} + \frac{B_1}{2}\cos\frac{x}{2}$

$y''_{чн} = -\frac{A_1}{4}\cos\frac{x}{2} - \frac{B_1}{4}\sin\frac{x}{2}$

$-6\frac{A_1}{4}\cos\frac{x}{2} - 6\frac{B_1}{4}\sin\frac{x}{2} + \frac{A_1}{2}\sin\frac{x}{2} - \frac{B_1}{2}\cos\frac{x}{2} -$

$-A_1\cos\frac{x}{2} \quad B_1\sin\frac{x}{2} = \cos\frac{x}{2}$

$\begin{cases} -\frac{3A_1}{2} - \frac{B_1}{2} - A_1 = 1 \\ -\frac{3B_1}{2} + \frac{A_1}{2} \cdot B_1 = 0 \end{cases}$ $\quad \begin{cases} -5A_1 - B_1 = 2 \\ 5B_1 = A_1 \end{cases}$

$A_1 = \frac{-5}{13} \quad B_1 = \frac{-1}{13}$

$y_{чн_1} = \frac{-5}{13}\cos\frac{x}{2} - \frac{1}{13}\sin\frac{x}{2}$

3) $f_2 = e^{\frac{x}{2}} = e^{\frac{1}{2}x}(1\cdot\cos 0 + 0\cdot\sin 0)$

$\alpha = \frac{1}{2} \quad \beta = 0$

$n = 0 \quad m = 1$

$y_{чн_2} = xe^{\frac{1}{2}x}\cdot C$

$y'_{чн_2} = Ce^{\frac{1}{2}x} + \frac{1}{2}x\cdot e^{\frac{1}{2}x}\cdot C$

$y''_{чн_2} = \frac{1}{2}Ce^{\frac{1}{2}x} + \frac{1}{2}Ce^{\frac{1}{2}x} + \frac{1}{4}Cxe^{\frac{1}{2}x} = Ce^{\frac{1}{2}x} + \frac{1}{4}Cxe^{\frac{1}{2}x}$

$$6Ce^{\frac{1}{2}x} + \frac{6}{4}Cxe^{\frac{1}{2}x} - Ce^{\frac{1}{2}x} - \frac{1}{2}Cxe^{\frac{1}{2}x} - Cxe^{\frac{1}{2}x} = e^{\frac{1}{2}x}$$

$$6C + \cancel{\frac{3Cx}{2}} - C - \cancel{\frac{1}{2}Cx - Cx} = 1$$

$$5C = 1$$

$$C = \frac{1}{5}$$

$$y_{ч.н_2} = \frac{1}{5}xe^{\frac{x}{2}}$$

③ $$y_{о.н} = C_1e^{-\frac{1}{3}x} + C_2e^{\frac{1}{2}x} - \frac{5}{13}\cos\frac{x}{2} - \frac{1}{13}\sin\frac{x}{2} + \frac{1}{5}xe^{\frac{x}{2}}$$

54

$$4y'' + 4y' + y = x^3 + 6x^2$$

$x=0$
$y=2$
$y'=0$

① $$4y'' + 4y' + y = 0$$

$$4k^2 + 4k + 1 = 0$$

$$k_{1,2} = -\frac{1}{2}$$

$$y_{о.о} = (C_1x + C_2)e^{-\frac{1}{2}x}$$

② $$4y'' + 4y' + y = x^3 + 6x^2$$

$$f(x) = x^3 + 6x^2 = e^0(x^3 + 6x^2)$$

$\alpha = 0 \quad \beta = 0$

$n = 3 \quad m = 0$

$$y_{ч.н} = A_1x^3 + A_2x^2 + A_3x + A_4$$

$$y'_{ч.н} = 3A_1x^2 + 2A_2x + A_3$$

$$y''_{ч.н} = 6A_1x + 2A_2$$

$$24A_1x + 8A_2 + 12A_1x^2 + 8A_2x + 4A_3 + A_1x^3 + A_2x^2 + A_3x + A_4 = x^3 + 6x^2$$

$$\begin{cases} 24A_1 + 8A_2 + A_3 = 0 \\ 8A_2 + A_4 + 4A_3 = 0 \\ A_1 = 1 \\ 12A_1 + A_2 = 6 \end{cases} \quad \begin{cases} A_3 = 48 - 24 \\ A_4 = 48 - 96 \\ A_1 = 1 \\ A_2 = -6 \end{cases} \quad \begin{cases} A_3 = 24 \\ A_4 = -48 \\ A_1 = 1 \\ A_2 = -6 \end{cases}$$

$$y_{ч.н} = x^3 - 6x^2 + 24x - 48$$

③ $y_{о.н} = (C_1 x + C_2) e^{-\frac{1}{2}x} + x^3 - 6x^2 + 24x - 48$

$x = 0$: $C_2 - 48 = 2$ $(y = 2)$

$C_2 = 50$

$(y' = 0)$

$C_1 - \frac{1}{2} C_2 = -24$

$C_1 = 1$

$$y_{ч.н} = (x + 50) e^{-\frac{1}{2}x} + x^3 - 6x^2 + 24x - 48$$

№5

$x^2 y'' + 4xy' + 2y = \sin x \quad |: x^2 (\neq 0) \qquad y_1 = \frac{1}{x}$

$$y'' + \frac{4y'}{x} + \frac{2y}{x^2} = \frac{\sin x}{x^2}$$

① $y'' + \frac{4y'}{x} + \frac{2y}{x^2} = 0$

$$y_2 = \frac{1}{x} \cdot \int \frac{e^{-\int \frac{4}{x} dx}}{\frac{1}{x^2}} dx = \frac{1}{x} \int e^{-4\ln|x|} x^2 dx =$$

$$= \frac{1}{x} \int \frac{dx}{x^2} = \frac{1}{x} \cdot \frac{-1}{x} = \frac{-1}{x^2}$$

$$y_{о.о} = C_1 \cdot \frac{1}{x} + C_2 \left(-\frac{1}{x^2}\right)$$

② $\begin{cases} C_1'(x) y_1(x) + C_2'(x) y_2(x) = 0 \\ C_1'(x) y_1'(x) + C_2'(x) y_2'(x) = \sin x / x^2 \end{cases}$

$$\begin{cases} \frac{C_1'}{x} - \frac{C_2'}{x^2} = 0 \\ -\frac{C_1'}{x^2} + \frac{2C_2'}{x^3} = \sin x / x^2 \end{cases} \qquad \begin{cases} x C_1' - C_2' = 0 \\ -x C_1' + 2C_2' = x \sin x \end{cases}$$

$$\begin{cases} C_2' = x\sin x & \Rightarrow \quad C_2 = -x\cos x + \sin x \\ C_1' = \sin x & \Rightarrow \quad C_1 = -\cos x \end{cases}$$

$$y_{ч.н} = (-x\cos x + \sin x)\left(-\frac{1}{x^2}\right) + \frac{-\cos x}{x} =$$

$$= -\frac{\sin x}{x^2}$$

(3) $$y_{о.н} = \frac{C_1}{x} + C_2\left(-\frac{1}{x^2}\right) - \frac{\sin x}{x^2}$$

X

№6.

$$\frac{dX}{dt} = AX \qquad A = \begin{bmatrix} 4 & 3 \\ 1 & 6 \end{bmatrix}$$

$$\begin{cases} \frac{dx_1}{dt} = 4x_1 + 3x_2 \\ \frac{dx_2}{dt} = x_1 + 6x_2 \end{cases} \qquad \begin{cases} \frac{d^2x_1}{dt^2} = 4\frac{dx_1}{dt} + 3\frac{dx_2}{dt} \\ \frac{dx_1}{dt} = 4x_1 + 3x_2 \end{cases}$$

$$\begin{cases} \frac{d^2x_1}{dt^2} = 16x_1 + 12x_2 + 3x_1 + 18x_2 \\ \frac{dx_1}{dt} = 4x_1 + 3x_2 \end{cases} \qquad \begin{cases} \frac{d^2x_1}{dt^2} = 19x_1 + 30x_2 \\ 3x_2 = \frac{dx_1}{dt} - 4x_1 \end{cases}$$

$$\frac{d^2x_1}{dt^2} - 10\frac{dx_1}{dt} + 21x_1 = 0$$

$$k^2 - 10k + 21 = 0$$

$$D = 16$$

$$k_1 = 3 \qquad k_2 = 7$$

$$x_1 = C_1 e^{3t} + C_2 e^{7t}$$

$$3x_2 = 3C_1 e^{3t} + 7C_2 e^{7t} - 4C_1 e^{3t} - 4C_2 e^{7t}$$

$$x_2 = -\frac{C_1}{3} e^{3t} + C_2 e^{7t}$$

Проверка:

$$\frac{dx_1}{dt} = 3C_1 e^{3t} + 7C_2 e^{7t}$$

$$4x_1 + 3x_2 = 4C_1 e^{3t} + 4C_2 e^{7t} - C_1 e^{3t} + 3C_2 e^{7t} = 3C_1 e^{3t} + 7C_2 e^{7t}$$

$$\frac{dx_2}{dt} = -C_1 e^{3t} + 7C_2 e^{7t}$$

$$x_1 + 6x_2 = C_1 e^{3t} + C_2 e^{7t} - 2C_1 e^{3t} + 6C_2 e^{7t} = -C_1 e^{3t} + 7C_2 e^{7t}$$

$$X = C_1 e^{3t} \begin{pmatrix} 1 \\ -\frac{1}{3} \end{pmatrix} + C_2 e^{7t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

N 2 16.05.05