7. $f(x)=\begin{cases}1+x & 0<x<1\\ 3-x & 1<x<3\end{cases}$ по синусам

$$b_n=\frac{2}{3}\cdot\left[\int_0^1(1+x)\cdot\sin\frac{n\pi x}{3}dx+\int_1^3(3-x)\cdot\sin\frac{n\pi x}{3}dx\right]=$$

$$=\frac{2}{3}\cdot\left[-(1+x)\cdot\frac{3}{\pi n}\cdot\cos\frac{n\pi x}{3}\Bigg|_0^1+\frac{3}{\pi n}\int_0^1\cos\frac{\pi nx}{3}dx-\frac{3}{\pi n}\cdot(3-x)\cdot\cos\frac{n\pi x}{3}\Bigg|_1^3-\right.$$

$$\left.-\frac{3}{\pi n}\int_1^3\cos\frac{n\pi x}{3}dx\right]=\frac{2}{3}\cdot\left[-\frac{6}{n\pi}\cdot\cos\frac{n\pi}{3}+\frac{3}{\pi n}+\frac{9}{\pi^2n^2}\cdot\sin\frac{n\pi x}{3}\Bigg|_0^1+\frac{6}{n\pi}\cdot\cos\frac{n\pi}{3}-\right.$$

$$\left.-\frac{9}{\pi^2n^2}\cdot\sin\frac{n\pi x}{3}\Bigg|_1^3\right]=\frac{2}{3}\cdot\left[\frac{3}{\pi n}+\frac{18}{\pi^2n^2}\cdot\sin\frac{n\pi}{3}\right]$$

Ответ: $f(x)=\sum\limits_{n=1}^{\infty}\frac{2}{\pi n}\cdot\left[1+\frac{6}{\pi n}\cdot\sin\frac{\pi n}{3}\right]\cdot\sin\frac{\pi nx}{3}$

Вариант 10.

1. $\sum\limits_{n=1}^{\infty}\frac{3^n\cdot\left(x-\frac{1}{3}\right)^n}{2^{2n}\cdot\left(\sqrt{n}-2^{-n}\right)}$

$$\lim_{n\to\infty}\frac{3^{n+1}\cdot\left|x-\frac{1}{3}\right|^{n+1}\cdot2^{2n}\cdot(\sqrt{n}-2^{-n})}{2^{2n+2}\cdot(\sqrt{n+1}-2^{-(n+1)})\cdot3^n\cdot\left|x-\frac{1}{3}\right|^n}=\frac{3}{2^2}\cdot\left|x-\frac{1}{3}\right|$$

$\left|x-\frac{1}{3}\right|<\frac{4}{3}$ $\boxed{-1\le x<\frac{5}{3}}$

$x-\frac{1}{3}=\frac{4}{3}$ $\sum\limits_{n=1}^{\infty}\frac{1}{\sqrt{n}-2^{-n}}$ - расходится

$x-\frac{1}{3}=-\frac{4}{3}$ $\sum\limits_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}-2^{-n}}$ - сходится условно

т.к. $\underset{n\to\infty}{a_n\to0}$ и

$$\frac{|a_{n+1}|}{|a_n|}=\frac{\sqrt{n}-\frac{1}{2^n}}{\sqrt{n+1}-2^{\frac{1}{n+1}}}=\sqrt{\frac{n}{n+1}}\cdot\left(\frac{1-\frac{1}{2^n\cdot\sqrt{n}}}{1-\frac{1}{2^{n+1}\cdot\sqrt{n+1}}}\right)<1$$

2. $\frac{3}{x^2-x-2}$ по степеням $(x+2)$

$$\frac{3}{(x+1)(x-2)}=-\frac{1}{x+1}+\frac{1}{x-2}$$

$$-\frac{1}{x+1}=\frac{-1}{(x+2)-1}=\frac{1}{1-(x+2)}=\sum_{n=0}^{\infty}(x+2)^n$$ $|x+2|<1$

$$\frac{1}{x-2}=\frac{1}{(x+2)-4}=-\frac{1}{4}\cdot\frac{1}{1-\frac{x+2}{4}}=-\frac{1}{4}\cdot\sum_{n=0}^{\infty}\frac{(x+2)^n}{4^n}\qquad |x+2|<4$$

$$\frac{3}{(x+1)(x-2)}=\sum_{n=0}^{\infty}\left(1-\frac{1}{4^{n+1}}\right)\cdot(x+2)^n\qquad |x+2|<1$$

$$\boxed{-3<x<-1}$$

3. $\int_0^2 e^{\frac{x^3}{12}}dx$

$$e^{\frac{x^3}{12}}=1+\frac{x^3}{12}+\frac{x^6}{12^2\cdot 2!}+\frac{x^9}{12^3\cdot 3!}+\dots+\frac{x^{3n}}{12^n\cdot n!}+\dots\qquad -\infty<x<+\infty$$

$$\int_0^2 e^{\frac{x^3}{12}}dx=2+\frac{2^4}{12\cdot 4}+\frac{2^7}{12^2\cdot 2!\cdot 7}+\frac{2^{10}}{12^3\cdot 3!\cdot 10}+\dots$$

$$|R|=\frac{2^{13}}{12^2\cdot 4!\cdot 13}+\frac{2^{16}}{12^5\cdot 5!\cdot 16}+\dots<\frac{2^{13}}{12^4\cdot 4!\cdot 13}\cdot\left(1+\frac{2^3}{12\cdot 5}+\frac{2^6}{12^2\cdot 5^2}+\dots\right)=$$

$$=\frac{2^{13}}{12^4\cdot 4!\cdot 13}\cdot\frac{1}{1-\frac{2^3}{60}}=\frac{2^2\cdot 60}{3^5\cdot 13\cdot(60-2^3)}<0{,}0002$$

	$2=2{,}000$		
+	$\frac{24}{12\cdot 4}=0{,}3333$	(+)	с недостатком
	$\frac{2^7}{12^2\cdot 2!\cdot 7}=0{,}0635$	(-)	с избытком
	$\frac{2^{10}}{12^3\cdot 3!\cdot 10}=0{,}0099$	(-)	с избытком
	2,4067		

$$2{,}4066<\int_0^2 e^{\frac{x^3}{12}}dx<2{,}4070$$

<u>**Ответ:**</u> $\int_0^2 e^{\frac{x^3}{12}}dx=2{,}407$

4. $y'=\sin x+\cos y\qquad y|_{x=\pi/2}=\frac{\pi}{2}$

$$y(x)=y\left(\frac{\pi}{2}\right)+y'\left(\frac{\pi}{2}\right)\cdot\left(x-\frac{\pi}{2}\right)+\frac{y''\left(\frac{\pi}{2}\right)}{2!}\cdot\left(x-\frac{\pi}{2}\right)^2+\frac{y'''\left(\frac{\pi}{2}\right)}{3!}\cdot\left(x-\frac{\pi}{2}\right)^3+\dots$$

$$y\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$$

$y'=\sin x+\cos y\qquad y'\left(\frac{\pi}{2}\right)=1$

$$y'' = \cos x - \sin y y' \qquad y''\left(\frac{\pi}{2}\right) = -1$$

$$y''' = -\sin x - \cos y \cdot (y')^2 - y'' \sin y \qquad y'''\left(\frac{\pi}{2}\right) = -\frac{\pi}{2} + \frac{\pi}{2} = 0$$

$$y(x) = \frac{\pi}{2} + \frac{1}{1!}\cdot\left(x - \frac{\pi}{2}\right) - \frac{1}{2!}\cdot\left(x - \frac{\pi}{2}\right)^2 + \dots$$

5. $(x^2 - 3)y'' - 2xy' + 2y = (5 - x^2)\sin x - 2x\cos x \qquad y(0) = 0 \qquad y'(0) = 1$

$$\begin{array}{r|l} 2 & y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_n x^n + \dots \\ -2 & xy'(x) = a_1 x + 2a_2 x^2 + 3a_3 x^3 + \dots + na_n \cdot x^n + \dots \\ -3 & y''(x) = 2a_2 + 3\cdot 2a_3 x + 4\cdot 3a_4 x^2 + 5\cdot 4a_5 x^3 + \dots + (n+2)(n+1)a_{n+2}x^n + \dots \\ 1 & x^2 y''(x) = 2a_2 x^2 + 3\cdot 2a_3 x^3 + \dots + n(n-1)a_n x^n + \dots \end{array}$$

$$(2a_0 - 6a_2) - 3\cdot 3\cdot 2\cdot a_3 x - 3\cdot 4\cdot 3a_4 x^2 + (2a_3 - 3\cdot 5\cdot 4a_5)x^3 + \dots +$$
$$+ [(n-1)(n-2)a_n - 3(n+2)(n+1)a_{n+2}]x^n + \dots$$

$$\begin{array}{r|l} 5 & \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^{n-1}\cdot\frac{x^{2n-1}}{(2n-1)!} + \dots \\ - & x^2 \sin x = x^3 - \frac{x^5}{3!} + \dots + (-1)^n \cdot \frac{x^{2n-1}}{(2n-3)!} + \dots \\ -2 & x\cos x = x - \frac{x^3}{2!} + \frac{x^5}{4!} - \dots + (-1)^{n-1}\cdot\frac{x^{2n-1}}{(2n-2)!} + \dots \end{array}$$

$$3x - \frac{5}{3!}x^3 + \frac{3}{4!}x^5 - \dots + (-1)^{n-1}\cdot\frac{4n^2 - 10n + 9}{(2n-1)!}x^{2n-1} + \dots$$

$$a_0 = 0 \qquad a_1 = 1$$

$$2a_0 - 6a_2 = 0 \qquad a_2 = 0$$

$$-3\cdot 3\cdot 2a_3 = 3 \qquad a_3 = -\frac{1}{3!}$$

$$-3\cdot 4\cdot 3a_4 = 0 \qquad a_4 = 0$$

$(n-1)(n-2)\cdot a_n + (n+2)(n+1)\cdot a_{n+2} = 0 \qquad n = 2m$ - четные $\qquad a_{2n} = 0$

$$(2n-2)(2n-3)\cdot a_{2n-1} + (2n+1)\cdot 2n\cdot a_{2n+1} = (-1)^{n-1}\cdot\frac{4n^2 - 10n + 9}{(2n-1)!}$$

Пусть $a_{2n-1} = (-1)^{n-1}\cdot\dfrac{1}{(2n-1)!}$

$$(2n-2)(2n-3)\cdot(-1)^{n-1}\cdot\frac{1}{(2n-1)!} - 3\cdot(2n+1)2na_{2n+1} = (-1)^{n-1}\cdot\frac{4n^2 - 10n + 9}{(2n-1)!}$$

$$a_{2n+1} = (-1)^{n-1}\cdot\frac{4n^2 - 10n + 9 - 4n^2 + 10n - 6}{(-3)\cdot(2n-1)!}$$

$$a_{2n+1} = (-1)^n\cdot\frac{1}{(2n-1)!}$$

$$y(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots + (-1)^n\cdot\frac{1}{(2n+1)!}x^{2n+1} + \dots$$

$$y\left(\frac{3}{4}\right)=\frac{3}{4}-\frac{1}{3!}\cdot\left(\frac{3}{4}\right)^3+\frac{1}{5!}\cdot\left(\frac{3}{4}\right)^5-\ldots+(-1)^n\cdot\frac{1}{(2n+1)!}\cdot\left(\frac{3}{4}\right)^{2n+1}+\ldots$$

$$|R|<\frac{1}{5!}\cdot\left(\frac{3}{4}\right)^5<0{,}002$$

$$\frac{3}{4}=0{,}750$$

$$-\frac{1}{3!}\cdot\left(\frac{3}{4}\right)^3=0{,}070 \quad (+) \qquad \text{с недостатком}$$

$$0{,}680$$

$$y\left(\frac{3}{4}\right)\approx 0{,}68$$

Ответ: $y(x)=\sin x \qquad y\left(\frac{3}{4}\right)\approx 0{,}68$

6. $f(x)=\begin{cases}1 & -\pi<x<\dfrac{\pi}{2}\\ -1 & \dfrac{\pi}{2}<x<\pi\end{cases}$

$$a_0=\frac{1}{\pi}\cdot\left[\int_{-\pi}^{\pi/2}dx-\int_{\pi/2}^{\pi}dx\right]=1$$

$$a_n=\frac{1}{\pi}\cdot\left[\int_{-\pi}^{\pi/2}\cos nxdx+\int_{\pi/2}^{\pi}-\cos nxdx\right]=\frac{1}{\pi}\cdot\left[\frac{1}{n}\cdot\sin nx\Big|_{-\pi}^{\pi/2}-\frac{1}{n}\cdot\sin nx\Big|_{\pi/2}^{\pi}\right]=\frac{2}{\pi n}\cdot\sin\frac{\pi n}{2}$$

$$b_n=\frac{1}{\pi}\cdot\left[\int_{-\pi}^{\pi/2}\sin nxdx-\int_{\pi/2}^{\pi}\sin nxdx\right]=\frac{1}{\pi}\cdot\left[-\frac{1}{n}\cos nx\Big|_{-\pi}^{\pi/2}+\frac{1}{n}\cos nx\Big|_{\pi/2}^{\pi}\right]=$$

$$=\frac{2}{\pi n}\cdot\left[(-1)^n-\cos\frac{n\pi}{2}\right]$$

Ответ: $f(x)=\frac{1}{2}+\frac{2}{\pi}\cdot\sum_{n=1}^{\infty}\frac{1}{n}\cdot\sin\frac{n\pi}{2}\cdot\cos nx+\frac{1}{n}\cdot\left[(-1)^n-\cos\frac{n\pi}{2}\right]\cdot\sin nx$

7. $f(x)=\begin{cases}1+x & 0<x<1\\ 3-x & 1<x<3\end{cases}$ по косинусам

$$a_0=\frac{2}{3}\cdot\left[\int_0^1(1+x)dx+\int_1^3(3-x)dx\right]=\frac{7}{3}$$

$$a_n=\frac{2}{3}\cdot\left[\int_0^1(1+x)\cdot\cos\frac{\pi nx}{3}dx+\int_1^3(3-x)\cos\frac{\pi nx}{3}dx\right]=$$

$$=\frac{2}{3}\cdot\left[\frac{3}{n\pi}\sin\frac{\pi nx}{3}+\frac{3x}{\pi n}\cdot\sin\frac{\pi nx}{3}+\frac{9}{n^2\pi^2}\cos\frac{\pi nx}{3}\right]\Bigg|_0^1+$$

$$+\left[\frac{9}{n\pi}\sin\frac{\pi nx}{3}-\frac{3x}{\pi n}\cdot\sin\frac{\pi nx}{3}-\frac{9}{n^2\pi^2}\cdot\cos\frac{\pi nx}{3}\right]\Bigg|_1^3=\frac{6}{n^2\pi^2}\cdot\left[2\cos\frac{n\pi}{3}-1+(-1)^{n+1}\right]$$

Ответ: $f(x)=\frac{7}{6}+\frac{6}{\pi^2}\cdot\sum_{n=1}^{\infty}\frac{1}{n^2}\cdot\left[2\cos\frac{\pi n}{3}-1+(-1)^{n+1}\right]\cdot\cos\frac{\pi n x}{3}.$

Вариант 11.

1. $\sum_{n=1}^{\infty}\frac{4^n}{n^4+4^{-n}}\cdot\left(x+\frac{3}{2}\right)^{2n-1}$

$$\lim_{n\to\infty}\frac{4^{n+1}\cdot\left|x+\frac{3}{2}\right|^{2n+1}\cdot(n^4+4^{-n})}{[(n+1)^4+4^{-(n+1)}]\cdot\left|x+\frac{3}{2}\right|^{2n-1}\cdot 4^n}=4\cdot\left|x+\frac{3}{2}\right|^2; \qquad \left|x+\frac{3}{2}\right|<\frac{1}{2}$$

$x+\frac{3}{2}=\frac{1}{2}$ $\qquad \sum_{n=1}^{\infty}\frac{2}{n^4+4^{-n}}$ сходится

$x+\frac{3}{2}=-\frac{1}{2}$ $\qquad \sum_{n=1}^{\infty}\frac{(-1)^n\cdot 2}{n^4+4^{-n}}$ сходится

Ответ: $-2\le x\le -1$

2. $\cos^2 x$ $\qquad a=\frac{\pi}{3}$

$$\cos^2 x=\frac{1}{2}+\frac{1}{2}\cos 2x$$

$$\cos 2x=\cos 2\cdot\left[\left(x-\frac{\pi}{3}\right)+\frac{\pi}{3}\right]=\cos\left[2\cdot\left(x-\frac{\pi}{3}\right)+\frac{2\pi}{3}\right]=\cos\frac{2\pi}{3}\cdot\cos 2\cdot\left(x-\frac{\pi}{3}\right)-$$

$$-\sin\frac{2\pi}{3}\cdot\sin 2\cdot\left(x-\frac{\pi}{3}\right)=-\frac{1}{2}\cos 2\cdot\left(x-\frac{\pi}{3}\right)-\frac{\sqrt{3}}{2}\sin 2\cdot\left(x-\frac{\pi}{3}\right)$$

Ответ:

$$\cos^2 x=\frac{1}{2}-\frac{1}{4}\cdot\sum_{n=0}^{\infty}(-1)^n\cdot\left[\frac{2^{2n}\cdot\left(x-\frac{\pi}{3}\right)^{2n}}{(2n)!}+(?)+\frac{2^{2n+1}\cdot\left(x-\frac{\pi}{3}\right)^{2n+1}}{(2n+1)!}\right],\quad -\infty<x<\infty$$

3. $\int_0^{0,5}\frac{dx}{\sqrt[5]{1+x^3}}$

$$\frac{1}{\sqrt[5]{1+x^3}}=(1+x^3)^{-\frac{1}{5}}=1-\frac{1}{5}x^3+\frac{1\cdot 6}{5^2\cdot 2!}x^6-\frac{1\cdot 6\cdot 11}{5^3\cdot 3!}x^9+\dots$$

$$\int_0^{0,5}\frac{dx}{\sqrt[5]{1+x^3}}=\frac{1}{2}-\frac{1}{5\cdot 4\cdot 2^4}+\frac{1\cdot 6}{5^2\cdot 2!\cdot 7\cdot 2^7}-\frac{1\cdot 6\cdot 11}{5^3\cdot 3!\cdot 10\cdot 2^{10}}+\dots$$

$$|R|<\frac{1\cdot 6}{5^2\cdot 2!\cdot 7\cdot 2^7}<0{,}0002 \qquad \int_0^{0,5}\frac{1}{\sqrt[5]{1+x^3}}\approx\frac{1}{2}-\frac{1}{5\cdot 4\cdot 2^4}\approx 0{,}497$$

4. $y'=\sin 2x+\cos y$ $\qquad y\left(\frac{\pi}{2}\right)=\pi$