Типавой расчет №1

по теоретической механике

„Динамика точки"

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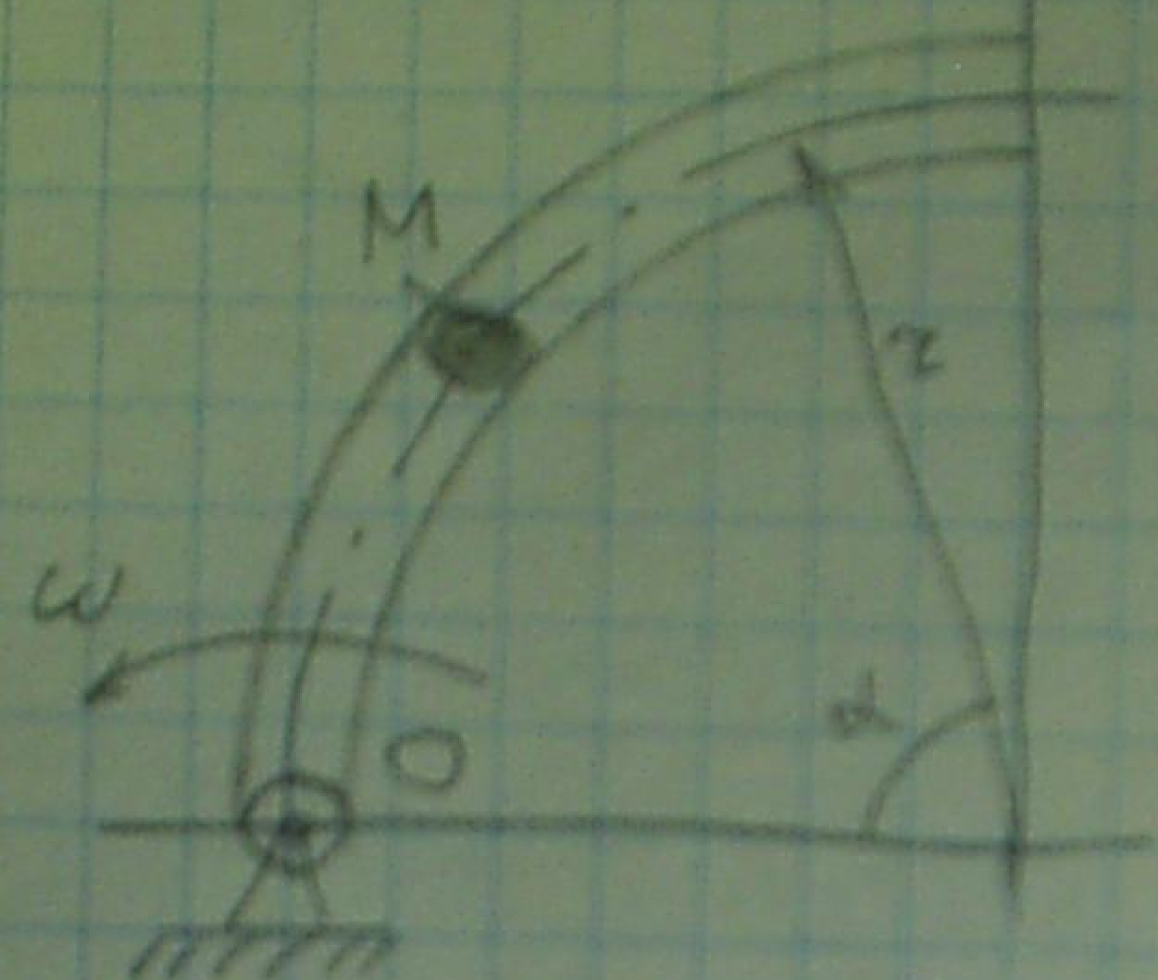
группы МТ4-31

10 Вариант



Масса $m = 1$ кг

$r = 0,2$ м

$\alpha = 90°$

$\omega = 5$ рад/с — const

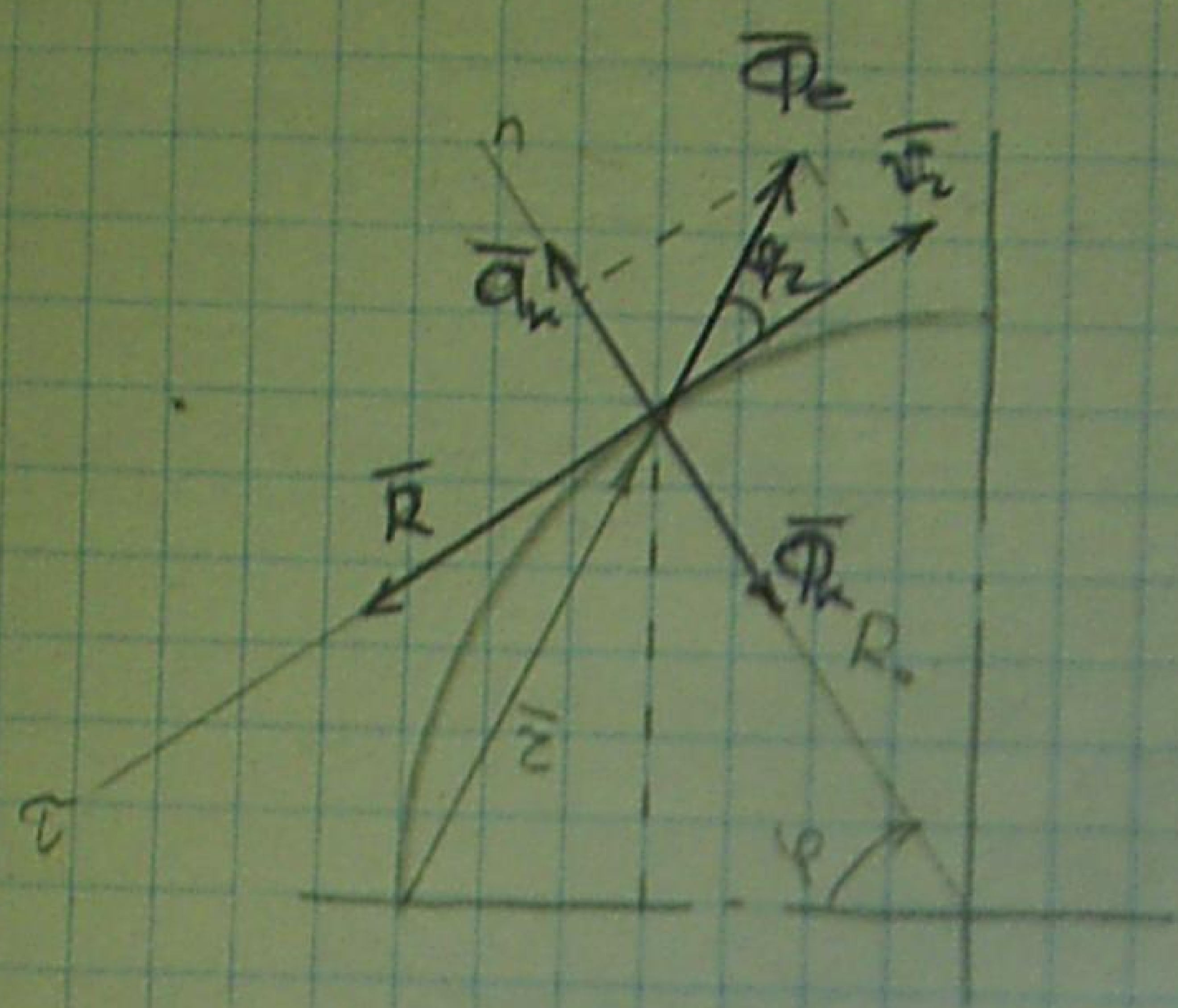
F-сопротивления $\overline{R} = -\mu |\overline{v}_2| \cdot \overline{v}_2$

$R = \mu v_2^2$ v_2 - относит. скор.

$\mu = 1,5 \cdot H \cdot с^2/м^2$

$M_0 \rightarrow$ к оси вращения.

Определить v_2 в момент вылета тела из трубки.



$$r^2 = (R_0 - R_0\cdot\cos\varphi)^2 + (R_0\cdot\sin\varphi)^2$$

$$r^2 = R_0^2 - 2R_0^2\cdot\cos\varphi + R_0^2\cos^2\varphi + R_0^2\cdot\sin^2\varphi$$

$$r^2 = R_0^2 - 2R_0^2\cos\varphi + R_0^2$$

$$r^2 = 2R_0^2(1-\cos\varphi) = 2R_0^2\left(\sin^2\frac{\varphi}{2} + \cos^2\frac{\varphi}{2} - \cos^2\frac{\varphi}{2} + \sin^2\frac{\varphi}{2}\right)$$

$$r^2 = 4R_0^2\sin^2\frac{\varphi}{2} \qquad \boxed{r = 2R_0\sin\frac{\varphi}{2}}$$

$$\tau:\quad m\overline{a_2^\tau} = \overline{\Phi_e}\cdot\cos\frac{\varphi}{2} + \overline{R} = \Phi_e\cdot\cos\frac{\varphi}{2} - R$$

$$\Phi_e = -a_e m = m\omega_0^2 r$$

$$\overline{R} = -\mu|\overline{v_2}|\overline{v_2} = \mu v_2^2$$

$$ma_2^\tau = m\omega_0^2 r\cdot\cos\frac{\varphi}{2} - \mu v_2^2$$

$$a_2^\tau = \omega_0^2\cdot 2R_0\cdot\sin\frac{\varphi}{2}\cdot\cos\frac{\varphi}{2} - \mu\frac{v_2^2}{2}$$

$$a_2^\tau = \omega_0^2\cdot R\cdot\sin\varphi - \mu\frac{v_2^2}{2}$$

$$a_2^\tau = \frac{dv_2}{dt}$$

$$\frac{d(v_2^2)}{2R\,d\varphi} = \omega^2 R\cdot\sin\varphi - \frac{\mu}{m}v_2^2$$

$$\frac{d(v_2^2)}{d\varphi} = 2\omega R^2\sin\varphi - 2R\frac{\mu}{m}v_2^2$$

$$\boxed{\begin{array}{ll} y = v_2^2 & k_1 = 2\omega^2 R^2 \\ x = \varphi & k_2 = 2R\dfrac{\mu}{m} \end{array}}$$

$$\frac{dy}{dx} = k_1 \cdot \sin x - k_2 y$$

$$\underbrace{\frac{dy}{dx} = -k_2 y} \quad \text{– лин. однор}$$

$$\ln y - \ln C = -k_2 x \qquad y_{о.о} = C \cdot e^{-k_2 x} \qquad y_{о.н} = C(x)\, e^{-k_2 x}$$

$$y' = C'(x) \cdot e^{-k_2 x} - k_2\, C(x)\, e^{-k_2 x}$$

$$C'(x) e^{-k_2 x} \cancel{- k_2 C(x) \cdot e^{-k_2 x}} = k_1 \sin x \cancel{- k_2 C(x) \cdot e^{-k_2 x}}$$

$$C'(x) \cdot e^{-k_2 x} = k_1 \cdot \sin x$$

$$C'(x) = k_1 \cdot e^{k_2 x} \cdot \sin x$$

$$C = k_1 \int e^{k_2 x} \cdot \sin x\, dx \Rightarrow$$

$$\int e^{k_2 x} \cdot \sin x\, dx = \left\{ \begin{array}{l} du = e^{k_2 x} dx \\ u = \frac{1}{k_2} e^{k_2 x} \\ V = \sin x \\ dV = \cos x\, dx \end{array} \right\} = \frac{e^{k_2 x}}{k_2} \cdot \sin x - \frac{1}{k_2} \int e^{k_2 x} \cdot \cos x\, dx =$$

$$= \left\{ \begin{array}{l} du = e^{k_2 x} dx \\ u = \frac{1}{k_2} e^{k_2 x} \\ V = \cos x \\ dV = -\sin x\, dx \end{array} \right\} = \frac{e^{k_2 x}}{k_2} \cdot \sin x - \frac{e^{k_2 x}}{k_2^2} \cdot \cos x - \frac{1}{k_2^2} \int e^{k_2 x} \cdot \sin x\, dx.$$

$$\int e^{k_2 x}\sin x\,dx = \frac{e^{k_2 x}}{k_2}\cdot\sin x - \frac{e^{k_2 x}}{k_2^2}\cos x - \frac{1}{k_2^2}\int e^{k_2 x}\sin x\,dx$$

$$\underbrace{\left(1+\frac{1}{k_2^2}\right)}_{\frac{k_2^2+1}{k_2^2}}\int e^{k_2 x}\cdot\sin x\,dx = \frac{e^{k_2 x}}{k_2}\left(\sin x - \frac{1}{k_2}\cos x\right)$$

$$\int e^{k_2 x}\sin x\,dx = \frac{e^{k_2 x}(k_2\sin x - \cos x)}{k_2^2+1} + C_1$$

$$\Rightarrow \quad C = k_1\frac{e^{k_2 x}(k_2\sin x - \cos x)}{k_2^2+1} + C_1$$

$$y = C\cdot e^{-k_2 x} = \frac{k_1(k_2\sin x - \cos x)}{k_2^2+1} + C_1\cdot e^{-k_2 x}$$

Подставим Н. У.: $\boxed{\begin{array}{l} v_2 = 0 \Rightarrow v_2^2 = 0 \Rightarrow y = 0 \\ \varphi = 0 \Rightarrow x = 0 \end{array}}$

$$0 = k_1\frac{-1}{k_2^2+1} + C_1 \qquad C_1 = \frac{k_1}{k_2^2+1}$$

$$y = \frac{k_1(k_2\sin x - \cos x)}{k_2^2+1} + \frac{k_1}{k_2^2+1}\cdot e^{-k_2 x}.$$

$$\left.\begin{array}{l} y = v_2^2 \\ x = \varphi \end{array}\right\} \Rightarrow v_2^2 = \frac{k_1\cdot(k_2\cdot\sin\varphi - \cos\varphi + e^{-k_2\varphi})}{k_2^2+1}$$

$$k_1 = 2\omega^2 R^2 = 2\cdot 5^2\cdot(0{,}2)^2 = 2$$

$$k_1 = 2R\cdot\frac{\mu}{m} = 2\cdot 0{,}2\cdot\frac{1{,}5}{1} = 0{,}6 \qquad k_1^2 = 0{,}36$$

$$v_2^2 = \frac{2(0{,}6\cdot\sin\varphi - \cos\varphi + e^{-0{,}6\varphi})}{1{,}36}$$

$$v_2^2\Big/_{\varphi=\frac{\pi}{2}} = \frac{2(0{,}6 + e^{-0{,}3\pi})}{1{,}36} = \frac{2(0{,}6+0{,}3896)}{1{,}36} =$$

$$= \frac{2\cdot 0{,}9896}{1{,}36} = 1{,}455344026$$

$$v_2 = \sqrt{1{,}455344026} = 1{,}206\ \text{м/с}$$

Ответ: 1,206 м/с